Math 10B with Professor Stankova
Worksheet, Discussion #4; Tuesday, 2/5/2019
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Binomial Identities

Concepts

1. We can write $C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$. One basic identity we have is the **binomial theorem** which says
$$(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k.$$
There are other equalities that can be proven either algebraically or combinatorially; by counting the same team making strategy in two different ways.

Examples

2. Show that $\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$.

3. Prove that $\sum_{k=0}^{n} \binom{n}{k} = 2^n$.

Problems

4. True False $\displaystyle\sum_{k=1}^{100} k\binom{100}{k} = 100 \cdot 2^{99}$.

5. Prove that $\sum_{k=0}^{n} 2^k \binom{n}{k} = 3^n$.

6. What is the coefficient of x^2y^3 in $(2x-3y)^5$?

7. Prove that $k\binom{n}{k} = n\binom{n-1}{k-1}$ in two different ways.

8. What is the coefficient of x^4y^9 in $(2x^2+5y^3)^5$?

9. (Challenge) What is the coefficient of $x^2y^2z^2$ in $(x+y+z)^6$?

Permutations and Combinations

Examples

10. How many ways can 6 people play in 3 tennis matches if the matches occur at different times? If they occur at the same time (and are indistinguishable)?

Problems

11. How many ways are there to rearrange the letters of ZYZZYX?

12. How many ways can we distribute 12 different cookies to 3 people if each person gets 3 (there are 3 left over)?

13. How many ways can we separate 12 different cookies into 4 piles of 3 if the piles are indistinguishable?